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COMMENT

Comment on 'Duality in fractals'

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Abstract. The dual of the Sierpinski gasket is argued not to be a Bethe lattice.

In the paper of Nencka-Ficek (1985) it is proposed that the dual of a Sierpinski gasket is a Bethe lattice on the basis of a number of conditions defining the duality construction. However these conditions do not define the dual as commonly employed in statistical mechanics; indeed, condition (iii) that $\dim(E) = \dim(D) + \dim(\tilde{D})$, where E is the dimension of the embedding space and D and $\tilde{D}$ are respectively the fractal dimensions of the lattice and its dual, is not obeyed by duality constructions on regular lattices. For planar lattices the essential construction is (Biggs 1976, Savit 1980) that a lattice, L, possesses a dual, $\tilde{L}$, constructed from L by placing vertices of $\tilde{L}$ inside every elementary loop of L and connecting them with bonds such that every bond of $\tilde{L}$ cuts one bond of L. It is essential for the duality mapping of Hamiltonians that every vertex of one lattice is surrounded by an elementary cycle on the other (Savit 1980). Dhar (1981) first constructed the duals of fractal graphs. The dual of the Sierpinski gasket is illustrated in Melrose (1983) along with several other fractal duals. On fractals the duality construction can introduce a range of edge lengths on the dual lattice and employing the intrinsic lattice metric one finds (Melrose 1983) that the intrinsic dimensions of a lattice and its dual are related by $\tilde{D} = D/Q$, where Q is the connectivity of the original planar lattice; $\tilde{D}$ may exceed that of the plane, for the gasket $\tilde{D} \to \infty$. These results emphasise the graphical aspects of duality.

References

Biggs N L 1976 *London Math. Soc. Lecture Note series* **30** (Cambridge: Cambridge University Press)
Dhar D 1981 *J. Phys. A: Math. Gen.* **14** L185
Melrose J R 1983 *J. Phys. A: Math. Gen.* **16** L407
Nencka-Ficek H 1985 *J. Phys. A: Math. Gen.* **18** 327
Savit R 1980 *Rev. Mod. Phys.* **52** 453

